Link Models for Networks with Dynamic Topologies

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ABSTRACT

Dynamic hierarchical networks represent an architectural strategy for employing adaptive behavior in applications sensitive to highly variable external demands or uncertain internal conditions. The characteristics of such architectures are described, and the significance of adaptive capability is discussed. The necessity for assessing the tradeoffs between performance improvements (reduced end-to-end message transmission time, increased throughput) and the added costs (reconfiguration delays, redundant links, etc.) leads to the use of complex queueing models. The assumptions underlying the general model are stated, and a class of applicable models (queues in random environments or RE-queues) is introduced. Matrix-geometric methods are reviewed in terms of their suitability for addressing several variations of a subclass of RE-queue models. Matrix-geometric techniques are considered to offer the greatest promise for obtaining usable results for assessing the cost/benefit tradeoffs.

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1. Introduction

This paper addresses the analysis of a class of computer communication networks whose members are referred to as *dynamic hierarchical networks,* or *dynamic hierarchies.* The dynamic hierarchy is an architectural concept that represents a generalization of the conventional tree structured architecture in which the network operates under a centralized, hierarchical mode of control. An overriding characteristic of these conventional (static) hierarchies is that, at the root of a tree-structured topology, there exists a single node that exercises primary control. Secondary capabilities filter down through the remainder of the network in a hierarchical manner. With this basic characteristic in force at all times, if the topology is allowed to vary, the resultant network is a dynamic hierarchical network.

Section 2 contains further description of the dynamic hierarchy as well as discussions of its significance and the authors' previous research on the dynamic hierarchy capacity assignment problem. Section 3 presents various queueing models of potential applicability to analysis of the dynamic hierarchy. (Portions of Sections 2.1 and 3.1 have appeared elsewhere [NANR85]). The network model establishes a framework in which subsequent, single server models are introduced. Two variants of the $M/M/1$ queue in a random environment (e.g, see [EISM63], [YECU71], [NEUM81]) are proposed as models of links in the dynamic hierarchy. Discussion of these models is preceded by a summary of the theory from [NEUM81] applicable to the study of $M/M/1$ queues in random environments. Conclusions and comments on future research are contained in Section 4.

2. Dynamic Hierarchical Networks

2.1. Description

A dynamic hierarchy is suitable for an application that is sensitive to multiple external situations or seeks to adapt to changing internal conditions. Each external situation or collection of internal conditions defines a *scenario.* For each scenario, an apex node (and a corresponding hierarchical topology) is designated as the most beneficial. At any instant, the network conforms to one of the specified topologies. When the scenario changes, the network undergoes a transition, with the appropriate node becoming the apex of the hierarchy corresponding to the reconfigured topology.

In general, a dynamic hierarchy with K nodes has $L \geq K-1$ physical interconnections (links). In each configuration, there exist $K-1$ active links (whose traffic rates are nonzero) and $L-K+1$ inactive links (which carry no messages and thus have traffic rates of zero). The set of active links is such that the interconnections form a tree structured topology. A link is inaccessible and may be considered nonexistent in any configuration in which it is inactive.

Different configurations generally have different sets of active and inactive links. A link may be active in every configuration, but no link may be inactive in every configuration.

Varying sets of active and inactive links contribute to a network topology that is physically variable. As scenario changes occur, various links are logically enabled or inhibited, which, together with changes in the apex node, induce a dynamic topology. A network of this type may also be viewed as one with a varying set of redundant interconnections. The need for this redundancy may be motivated by constraints on network performance measures such as survivability, reliability, and response time.

Note that in the case where $L = K - 1$, the network topology is physically static. Every link is active in all configurations. The interconnections thus remain fixed. However, the network topology is logically variable as a result of changes in the apex node (and the corresponding changes in the hierarchical distribution of primary control). If this simple connectivity satisfies the requirements of a given application, such a network enables the realization of the benefits of the dynamic hierarchy without the added cost of redundant links.

Figures 1 and 2 contain examples which should clarify the difference between physically and logically dynamic topologies. Let A_i and I_i be the sets of active links and inactive links, respectively, in configuration i, $i = 1, 2, \ldots, M$, where M is the number of allowable configurations.

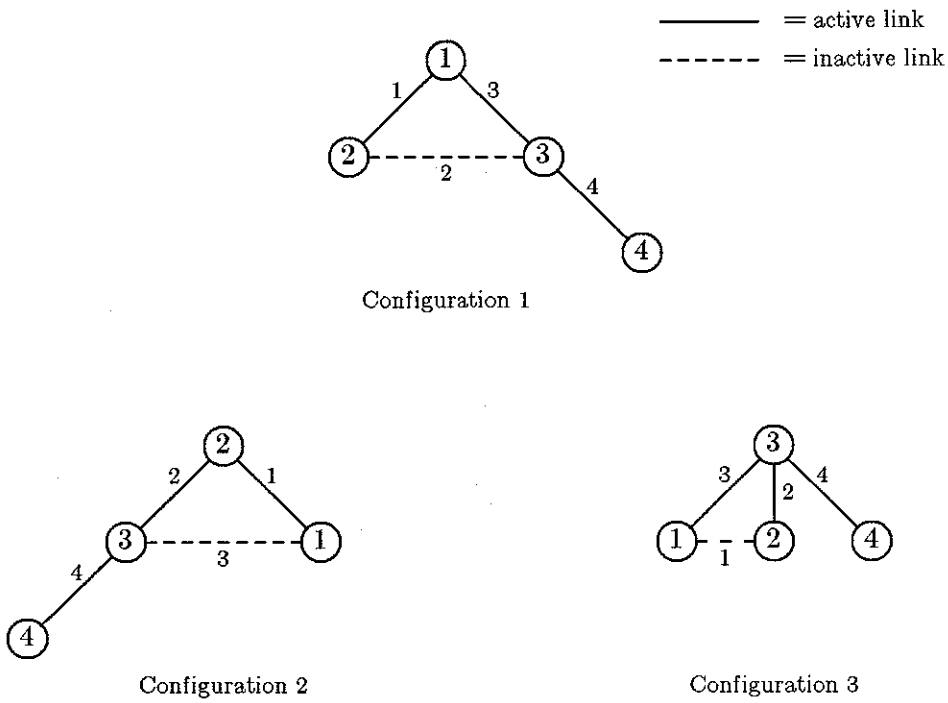

Figure 1. Example dynamic hierarchy with a physically varying topology

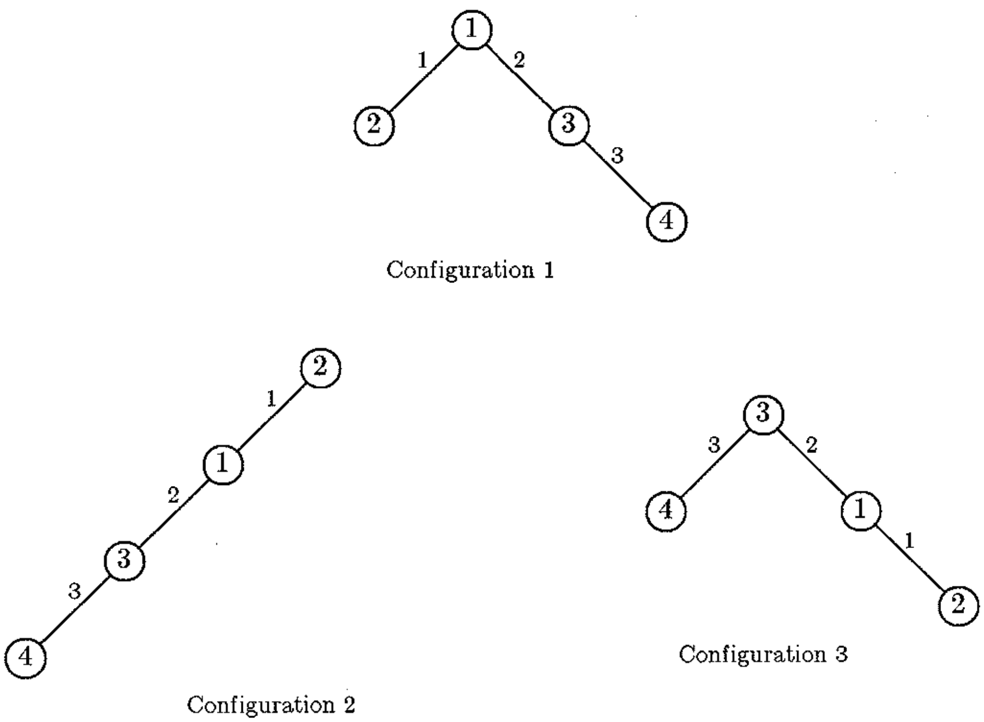

Figure 2. Example dynamic hierarchy with a logically varying topology

Both example networks have

$K = 4$ nodes and

$M = 3$ configurations.

The network of Figure 1, which exhibits a physically varying topology, has

$L = 4,$
$A_1 = \{1, 3, 4\}, I_1 = \{2\}$
$A_2 = \{1, 2, 4\}, I_2 = \{3\}$
$A_3 = \{2, 3, 4\}$, and $I_3 = \{1\}$

As required, the elements of A_i produce a tree structured topology for each configuration i.

Figure 2 contains a dynamic hierarchy whose topology is logically variable. This network has

$L = 3$ and

$A_i = \{1, 2, 3\}$ and $I_i = \phi,\ i = 1, 2, 3.$

The foregoing characterization of dynamic hierarchical networks is not meant to restrict the class of network topologies included. Although point-to-point, tree-structured topologies are of primary interest, this characterization applies equally well to other topological classes. If, for example, common bus topologies are of interest, the network has only one physical link. Such a network, subject to changing external situations, can be analyzed as a network with a logically variable topology by considering logical links, with effective capacities, between appropriate node pairs. In this case, variability results from defining logical links between node pairs.

The configurations of a possible bus-based counterpart of the network of Figure 1 are depicted in Figure 3. The hierarchical control structures corresponding to the topologies of Figure 1 are effected by defining logical links between the appropriate node pairs. Each pair connected by an active link in the point-to-point network utilizes a logical transmission path (represented by a dotted line in Figure 3) in the bus network.

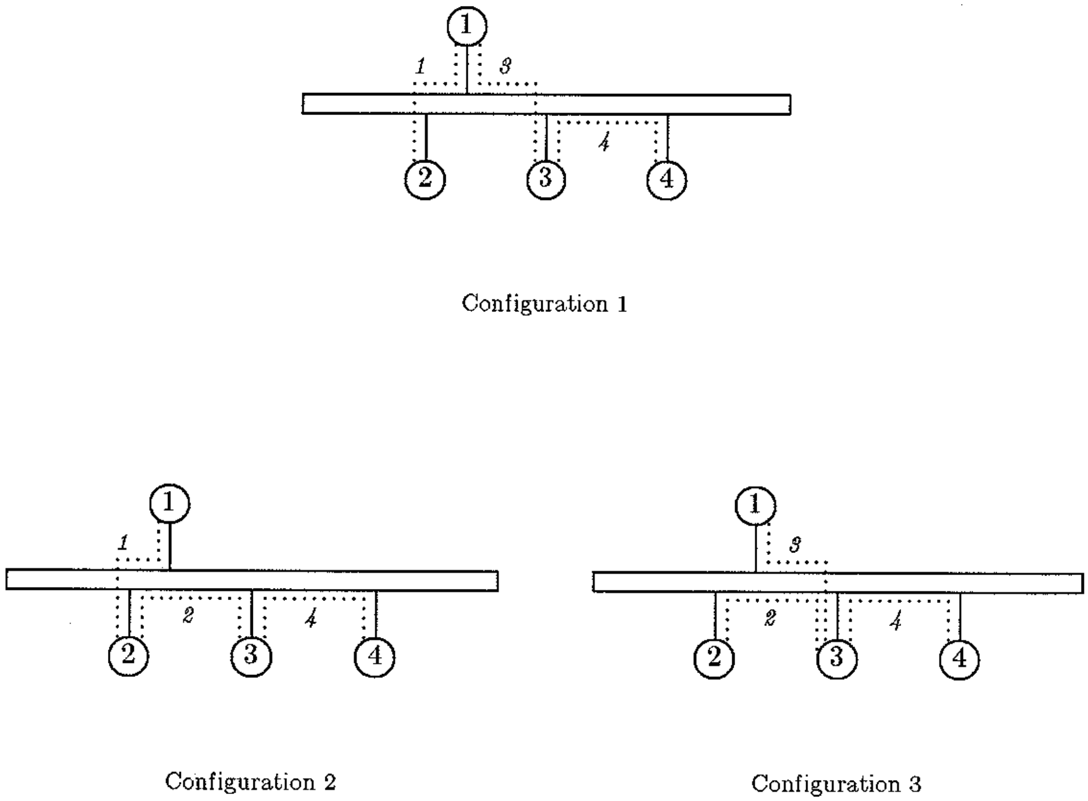

Figure 3. Example bus-based dynamic hierarchy

Consider a link j in a physically dynamic hierarchy and let j be inactive in at least one configuration. Then for each configuration i in which link j is active, the arrival rate of messages to j is some positive number $\lambda_j^{(i)}$. For each configuration in which link j is inactive, no traffic is routed to j and thus the arrival rate to j is zero.

For a bus-based dynamic hierarchy, it must be assumed that in each configuration, each node pair that communicates directly over a logical link is guaranteed a predetermined effective capacity by the protocol. The analysis depends fundamentally on the existence of fixed capacity transmission lines between specific node pairs. If the protocol does not provide these capacities, the analytic results do not hold. Suppose then that these capacities are available. Then a logical link j that exists in configuration i has some positive arrival rate $\lambda_j^{(i)}$. For the analysis, if link j does not exist in some configuration, then it is considered to exist conceptually but to have an arrival rate of zero.

Neither physically dynamic topologies nor bus and ring topologies are addressed directly in this paper. The model of the logically dynamic, point-to-point hierarchy applies to some extent to physically dynamic, point-to-point hierarchies and bus- and ring-based hierarchies †. However, the authors expect the analytic results to be less accurate when applied to networks in these topological classes.

2.2. Significance and Motivations

2.2.1. Application to the Physical Problem

Two separate but related considerations establish the dynamic hierarchy as a significant concept. First, a review of the literature indicates that local area networking, distributed computing, and combinations of the two will continue to experience increases in popularity and applicability. The dynamic hierarchy is intended as a means for providing local area networking and distribution of primary control in a particular application area. This area is characterized by the need for real-time or time-critical response and a desire to improve network survivability, reliability, availability, or other performance measures by employing a distributed mode of network operation while maintaining some form of global control. A reluctance or an inability to allow fully autonomous operation of the network elements may be due to the existence of natural control partitionings or a hierarchical command and control structure in the proposed application environment. Global control with limited distribution may be achieved through the use of the dynamic

† Protocol issues and other practical problems of network operation should be considered separately for each topological class.

hierarchy.

Primary advantages of this limited distribution of control include an increased ability to meet time-critical response requirements and a higher degree of flexibility in handling the variability of traffic patterns (arrival rates). Reconfiguration brings critical links and nodes closer to the apex of the hierarchy, thus increasing the ability of these resources to provide service in a timely manner. Higher flexibility results from choosing apex nodes and configurations that are best able to meet constraints (timing or otherwise) imposed by the different scenarios. The network need not operate under a static topology and control structure resulting from a compromise design to render adequate performance over all scenarios.

2.2.2. Theoretical Interest

The second supporting consideration derives from the work of Neuts [NEUM77,NEUM78,NEUM81] (and other papers), Purdue [PURP74,PURP78] (and other papers), and others [EISM63], [SCOM64], [YECU71,YECU73] on queues in random environments (*RE-queues*). Typical applications of the theory established therein include the modeling of systems that experience rush hour behavior in their arrival streams and of service counters that are subject to server breakdowns or rest periods. These and similar characteristics are not easily included in a traditional single server queueing model. The RE-queue represents a more detailed and realistic model in such cases.

Outside of the authors' preliminary research (e.g, [NANR87]), a network generalization of the RE-queue has not yet been proposed. Dynamic hierarchy protocols are discussed in [NAGS86] but the analysis therein is not queue-theoretic. An alternate analytic model and the treatment of *reconfiguration periods* are introduced in [BHAU86].

The research in progress involves modeling the dynamic hierarchy as a network of

$M/M/1$ queues in a random environment. Queue-theoretic results of this research will contribute significantly to a basis for understanding the performance characteristics of dynamic hierarchical (computer communication) networks. Further, the authors believe that these results may be applied with equal validity to systems other than computer communication networks. For example, just as there exist systems such as bank tellers and traffic lights, which experience rush hour arrival patterns and are reasonably modeled as RE-queues, systems such as transportation networks with seasonal demand levels and plant control networks with variable production rates also exists, for which a network of $M/M/1$ queues in a random environment may represent a reasonable model. Thus, the set of potential beneficiaries of this research includes both computer network analysts and others whose interests are outside (but possibly related to) the field of computer networks.

2.3. Previous Research

The authors' previous work on the dynamic hierarchy link capacity assignment problem is reported in [MOOR83], [NANR85], [NANR87]. First, an approximation of mean network delay (sojourn time) is derived. This approximation represents the basic measure of network performance for subsequent optimization and analysis. Next, a number of suboptimal, probabilistic and heuristic capacity assignment strategies are defined. Statistical techniques then are used to compare the effects of these strategies and to identify the best strategies.

For conventional networks, given the assumptions of Kleinrock [KLEL64], a closed form expression for mean delay is derived through the application of elementary queueing theory. This expression is extended as follows to provide an approximate measure of delay in the dynamic hierarchy: Let $\pi^{(i)}$ be the stationary probability of occurrence of configuration i. Now consider configuration i as if it were the topology of a static network and let $T^{(i)}$ be mean delay (as derived by Kleinrock) for that network. Mean delay for the dynamic hierarchy is approximated by taking the weighted sum, where the $\pi^{(i)}$ are the weights, of individual configuration mean delay. That is, T, approximate mean delay, is given by

$$T = \sum_{i=1}^{M} \pi^{(i)} T^{(i)}.$$

The dynamic hierarchy capacity assignment problem is:

> Given the set of configurations, the configuration probabilities, and a characterization of network traffic, select link capacities that minimize total cost subject to an upper bound on mean delay.

Two sets of strategies are created to assign capacities. Members of the first set, referred to as the probabilistic strategies, are created by adapting previous analytic results from conventional network design. Consider again configuration i as if it were the topology of a static network. Let $C_j^{(i)}$ be a capacity assignment for link j that is in some sense optimal for this network (for example, the $C_j^{(i)}$ might be calculated through the cost minimization counterpart of Kleinrock's square root strategy [KLEL76, p. 350]). Then the capacity assignment for link j in the dynamic hierarchy is

$$C_j = \sum_{i=1}^{M} \pi^{(i)} C_j^{(i)}.$$

Different formulae for the $C_j^{(i)}$ and variants of the weighted sum construction yield different assignment strategies.

Members of the second set, referred to as the heuristic strategies, are algorithmic in nature and assume a discrete cost function. These strategies are constructed by first defining a group of capacity assignment heuristics and then taking various combinations of these heuristics to produce composite capacity assignment algorithms. This approach follows the method of Maruyama, et al. in, for example, [MARK76].

As the final step, the strategies are compared through the use of analysis of variance

(ANOVA) procedures. Statistical comparisons are necessary because the the strategies are approximate and heuristic in nature and the need exists to extend the conclusions of any comparisons to all dynamic hierarchies. These tests are reported fully in [NANR87].

3. Queueing Models

3.1. Network Model

The ideal goal in analyzing the dynamic hierarchy is to derive theoretically exact solutions for queue length probabilities and mean waiting time from an appropriate network model. An *RE-network* (a network of $M/M/1$ queues in a random environment) is a natural candidate. The exact analysis of such a model is beyond the scope of this research. However, to provide a foundation for subsequent discussions, a description of the basic network model follows. This description assumes that network configuration changes are instantaneous, an assumption that is dropped in one of the single link models.

The assumptions that are necessary to model the dynamic hierarchy as an RE-network include the following:

(1) Scenarios changes occur according to a random (Markovian) environment process.

(2) The network configuration for each scenario is given.

(3) When the environment process is in state i, messages with source node j and destination node k arrive at node j according to a Poisson process with rate $\gamma_{jk}^{(i)}$.

(4) Given the state of the environment process, the arrival processes of (3) are mutually independent.

(5) Nodal processing times are negligible.

(6) Messages pass through the network in a store-and-forward fashion. That is, for each source/destination pair j, k in configuration i, there exists a uniquely specified shortest path $\psi_{jk}^{(i)}$ of links connecting nodes j and k. A message arriving at node j is alternately stored at the source or an intermediate node and transmitted across the next link in $\psi_{jk}^{(i)}$. Full receipt of the message is required prior to each forwarding operation. The message departs the network following its delivery to destination node k.

(7) At each node, there exists a separate first-come-first-served queue with unlimited buffer space for each outgoing link.

(8) Propagation times are negligible.

(9) Each node possesses sufficient processing capability to operate all incoming and outgoing links (transmissions) simultaneously.

(10) Links are physically capable of transmission in one direction only. Bidirectional transmission is enabled by connecting nodes with link pairs whose components transmit in opposite directions.

(11) Links are noiseless and error free.

(12) The lengths of arriving messages are exponentially distributed with mean μ^{-1} bits. Additionally, at each intermediate node in the path of a message, upon entering service, the length of the message is reset according to the same exponential distribution (A variant of Kleinrock's independence assumption [KLEL64]).

(13) The message length processes of (12) are mutually independent.

(14) The collections of processes of (3) and (12) are independent of each other.

Under these assumptions, definition of the appropriate stochastic process representing an RE-network is straightforward. Let $E = \{E(t);\ t \geq 0\}$, the environment process, be a stationary, irreducible Markov process with finite state space $S_E = \{1, 2, \ldots, M\}$, where M is the number of possible environments. Consider a network of L queues, each of which has infinite waiting room and serves customers according to a first-come-first-served discipline. Each queue $l \in \{1, 2, \ldots, L\}$ operates under the influence of the environment process as follows: On $\{E(t) = i\}$, $i \in S_E$,

(1) Arrivals from outside the network (external arrivals) occur according to a Poisson process with rate $\lambda_l'^{(i)}$.

(2) Service times are independent, identically distributed random variables following an exponential distribution with mean $\frac{1}{\mu C_l}$. (The mean message length is μ^{-1} bits and the transmission capacity is C_l bits/second).

Assume that the service time processes are mutually independent. Further, assume that given the state of E, the external arrival processes are mutually independent, and the collections of service time processes and arrival processes are independent of each other.

For routing purposes, messages (customers) are assumed to be typed according to their source and destination (nodes). A jk-message, a message with source j and destination k, arrives to node j, follows a fixed (unique for each environment state) path $\gamma_{jk}^{(i)}$ of links through the network, and departs after reaching node k.

Now let $\underset{\sim}{N} = \{\underset{\sim}{N}(t);\ t \geq 0\}$ be the process of network queue lengths, where

$$\underset{\sim}{N}(t) = (N_1(t), N_2(t), \ldots, N_L(t))$$

and $N_l(t)$ is the length (including the customer in service, if any) of queue l at time t. Then the process of interest, $(\underset{\sim}{N}, E) = \{\underset{\sim}{N}(t), E(t);\ t \geq 0\}$ is a Markov process with state space $(\bigtimes_{l=1}^{L} S_{N_l}) \times S_E$, where $S_{N_l} = \{0, 1, 2, \ldots\}$ is the state space of a single queue length process.

To model the dynamic hierarchy as above, the correspondence must be specified between the external arrival processes (in the form source/destination node pairs) and the resultant processes of external arrivals to the individual servers (links). Let $\Gamma^{(i)} = \{\gamma_{jk}^{(i)}\}$ be the matrix of arrival rates, expressed in terms of source/destination pairs, for configuration i. Consider a single node j with $\{l_1, l_2, \ldots, l_n\}$ the set of outgoing links that are connected to the node and are active in configuration i. For each $l \in \{l_1, l_2, \ldots, l_n\}$ define the external arrival rate to link l in configuration i as

$$\lambda_l'^{(i)} = \sum_{\{j,k:l\epsilon\psi_{jk}^{(i)}\}} \gamma_{jk}^{(i)}.$$

It can be shown that in configuration i, the external arrival process to link l is Poisson with rate $\lambda_l'^{(i)}$. Briefly, on $\{E(t) = i\}$, the original external arrival processes are independent, Poisson with rates $\gamma_{jk}^{(i)}$. Thus, the superposition of the specified processes is Poisson with rate $\lambda_l'^{(i)}$ [CINE75]. Further, given the state of the environment process, these individual link, external arrival processes are mutually independent and the collection of these processes is independent of the collection of service time (message length) processes. (Note that $\lambda_l'^{(i)}$ accounts for arrivals from outside the network only. The composite rate $\lambda_l^{(i)}$ of message arrivals to link l in configuration i includes contributions due to message forwarding inside the network. In general, derivation of the $\lambda_l^{(i)}$'s requires detailed knowledge of the steady state message flow rates).

3.2. Approximate Single Link Models

Since the solution to the RE-network is not expected to be of product form and the rates of flow in the network ar largely unknown, additional assumptions are necessary. In this paper, matrix-geometric methods are examined as a technique for analyzing the individual queues as if they were RE-queues in isolation. Research in progress, to be reported in a future paper, involves the introduction of an assumption that leads to simplified flow processes. The resultant simplified model and the single server results are used to derive RE-network performance measures such as mean waiting time and blocking (overflow) probability.

In modeling the dynamic hierarchy, nodal processing times are assumed negligible and each node is considered to possess sufficient processing capability to operate all incoming and outgoing links simultaneously. Thus, the links and their associated queues are the individual service systems of interest. Nodes are considered points at which messages are routed to the appropriate links or to outside of the network †. Two variants of the RE-queue are introduced to model these individual systems. The first variant incorporates the assumption that network configuration transitions (reconfigurations) are instantaneous. The second allows for a reconfiguration period preceding each transition.

(The following notational conventions are employed in this section: Symbols in bold type denote vectors. Depending on the context in which they appear, symbols in normal, non-bold type denote matrices or scalars. The symbols **0** and **1** denote the appropriate size vectors of zeros and ones, respectively).

3.2.1. Matrix-Geometric Results

Several main results from the RE-queue literature are needed to analyze the model of a dynamic hierarchy link in isolation. These results relate primarily to queue length and virtual waiting time. Since Neuts is the primary contributor to the theory of *phase processes*, or *PH-processes*, of which the RE-queue is an application, the presentation in this section follows the order and a modified form of the notation of his text on *matrix-*

† Nodes may also be considered message producers and consumers. However, the distinction between this view and the current view that messages originate outside the network and ultimately exit the network is not important with respect to this analysis.

geometric methods [NEUM81]. (The theory of *quasi-birth-and-death processes,* which are contained in the set of PH-based processes, originates with the dissertation of Wallace [WALV69]).

Neuts observes that in "the construction of algorithmic solutions [of stochastic models], it is overwhelmingly clear that the *structure* † is of paramount importance. The specific analytic form of the elements of the transition probability matrix is of far less consequence..." [NEUM81, p. 2]. The overriding characteristics of the matrix-geometric approach are

- Exploitation of structures of transition matrices or infinitesimal generators to reach solutions through probabilistic approaches. Transform methods are avoided when possible. In general, the transition matrices and generators are expressed in terms of sub-matrices that are defined by partitioning the original matrices. Most of the resultant solutions involve matrix operations.
- A "concern for solution methods that are implementable in a general and numerically stable manner and offer detailed information on at least some of the more complex models encountered in the study of practical queueing systems." [NEUM81, p. 3] This characteristic falls under the general heading of *computational probability,* which Neuts defines in the preface to [NEUM81]. The form of the (scalar) elements of a particular matrix is of much less importance than the ability to devise a robust algorithm to numerically solve one or more matrix equations.

A Markov process ‡ of the $GI/M/1$ type is one whose transition matrix $\tilde{Q}$ has the

† "Structure" refers to the structure as reflected in the transition matrix.
‡ Since continuous time Markov processes are of interest, this presentation skips the analogous theory for Markov chains.

form

$$\tilde{Q} = \begin{bmatrix} B_0 & A_0 & 0 & 0 & 0 & . & . & . \\ B_1 & A_1 & A_0 & 0 & 0 & . & . & . \\ B_2 & A_2 & A_1 & A_0 & 0 & . & . & . \\ B_3 & A_3 & A_2 & A_1 & A_0 & . & . & . \\ B_4 & A_4 & A_3 & A_2 & A_1 & . & . & . \\ . & . & . & . & . & . & & \\ . & . & . & . & . & & . & \\ . & . & . & . & . & & & . \end{bmatrix}$$

where the B_i and A_i are $M \times M$ nonnegative matrices. The Markov process $\tilde{Q}$ is assumed to be irreducible. $\tilde{Q}$ has state space $\{(i,\ j)\colon i \geq 0,\ 1 \leq j \leq M\}$, where the states are ordered lexicographically. The set $\{(i,\ 1),\ .\ .\ .\ ,\ (i,\ M)\}$ is called *level* i.

Let $\boldsymbol{x} = [\boldsymbol{x}_0,\ \boldsymbol{x}_1,\ \boldsymbol{x}_2, \ldots]$, where $\boldsymbol{x}_i = [x_i^{(1)} \ \cdots \ x_i^{(M)}]$ for each i, be the invariant probability vector of $\tilde{Q}$ and define

$$A = \sum_{k=0}^{\infty} A_k.$$

Under appropriate conditions (irreducible $\tilde{Q}$, nonsingular B_0 and A_1, and $A\mathbf{1} = \mathbf{0}$) Theorem 1.7.1 of Neuts [NEUM81, pp. 32-33], which is not repeated here, gives the stationary probability vector $\boldsymbol{x}$ satisfying $\boldsymbol{x}\tilde{Q} = \mathbf{0}$, $\boldsymbol{x}\mathbf{1} = 1$ for the general $GI/M/1$ type process.

When the generator $\tilde{Q}$ has a block tridiagonal structure, that is

$$\tilde{Q} = \begin{bmatrix} B_0 & A_0 & 0 & 0 & 0 & . & . & . \\ B_1 & A_1 & A_0 & 0 & 0 & . & . & . \\ 0 & A_2 & A_1 & A_0 & 0 & . & . & . \\ 0 & 0 & A_2 & A_1 & A_0 & . & . & . \\ 0 & 0 & 0 & A_2 & A_1 & . & . & . \\ . & . & . & . & . & . & & \\ . & . & . & . & . & & . & \\ . & . & . & . & . & & & . \end{bmatrix},$$

the $\tilde{Q}$ process is said to be a *quasi-birth-and-death process* or *QBD process*. In this case, the matrix A simplifies to

$$A = A_0 + A_1 + A_2.$$

The general theorem is restated as follows [NEUM81, pp. 82-83]:

> The process $\tilde{Q}$ is positive recurrent if and only if the minimal nonnegative solution R [the *rate matrix*] to the matrix-quadratic equation
>
> $$R^2 A_2 + R A_1 + A_0 = 0$$
>
> has all its eigenvalues inside the unit disk [denoted by $sp(R) < 1$] and the finite system of equations
>
> $$\boldsymbol{x}_0(B_0 + RB_1) = \mathbf{0}$$
> $$\boldsymbol{x}_0(I - R)^{-1}\mathbf{1} = 1$$
>
> has a unique positive solution $\boldsymbol{x}_0$.
>
> If the matrix A is irreducible, then $sp(R) < 1$ if and only if
>
> $$\boldsymbol{\pi} A_2 \mathbf{1} > \boldsymbol{\pi} A_0 \mathbf{1},$$
>
> where $\boldsymbol{\pi}$ is the stationary probability vector of A.
>
> The stationary probability vector $\boldsymbol{x} = [\boldsymbol{x}_0, \boldsymbol{x}_1, \ldots]$ of $\tilde{Q}$ is given by
>
> $$\boldsymbol{x}_i = \boldsymbol{x}_0 R^i, \text{ for } i \geq 0.$$
>
> The (equivalent) equalities
>
> $$RA_2\mathbf{1} - A_0\mathbf{1} = RB_1\mathbf{1} - B_0\mathbf{1} = 0$$
>
> hold.

Waiting times in queues modeled as QBD processes are analyzed as absorption times in finite Markov processes. The appropriate Markov process has generator

$$\tilde{Q}^0 = \begin{bmatrix} 0 & 0 & 0 & 0 & . & . & . \\ A_2 & D & 0 & 0 & . & . & . \\ 0 & A_2 & D & 0 & . & . & . \\ 0 & 0 & A_2 & D & . & . & . \\ . & . & . & . & . & & \\ . & . & . & . & & . & \\ . & . & . & . & & & . \end{bmatrix},$$

where A_2 and D are $M \times M$ matrices whose forms depend on the model under consideration.

Define $K = D^{-1}A_2$. Let $\boldsymbol{y}(0) = [\boldsymbol{y}_0(0), \boldsymbol{y}_1(0), \boldsymbol{y}_2(0), \ldots]$ be the initial probability vector of $\tilde{Q}^0$ and $W(x) = [W^{(1)}(x), W^{(2)}(x), \ldots, W^{(M)}(x)]$ be the vector of absorption probabilities, where $W^{(j)}(x) = P$(absorption into $(0, j)$ by time x). The primary results of interest here are [NEUM81, pp. 133-134]:

(1)The vector of Laplace-Stieltjes transforms of the components of $W(x)$ is

$$w(s) = \sum_{k=0}^{\infty} y_k(0)[(sI - D)^{-1}A_2]^k \text{ , for } s \geq 0 \text{and}$$

(2)The mean vector $-w'(0)$ is finite if and only if the vector $\sum_{k=1}^{\infty} k y_k(0)$ is finite. It is then given by

$$-w'(0) = \sum_{k=1}^{\infty} y_k(0) \sum_{\nu=0}^{k-1} K^{\nu}(-D^{-1})K^{k-\nu}.$$

Consider now an $M/M/1$ queue in a random environment. The environment is assumed to be an M state Markov process E with irreducible generator Q. On $\{E(t) = i\}$, arrivals occur according to a Poisson process with rate $\lambda^{(i)}$, service times are i.i.d. exponential random variables with mean $\frac{1}{\mu^{(i)}}$, and the arrival and service time processes are independent. The arrival and service rates change instantaneously when E changes state. The queueing discipline is first-come-first-served.

Let $N(t)$ be the queue length at time t. Define $\boldsymbol{\lambda} = [\lambda^{(1)}, \lambda^{(2)}, \ldots, \lambda^{(M)}]$ and $\boldsymbol{\mu} = [\mu^{(1)}, \mu^{(2)}, \ldots, \mu^{(M)}]$ as the arrival and service rate vectors, respectively. Then $(N, E) = \{(N(t), E(t));\ t \geq 0\}$ is a QBD process with generator

$$\tilde{Q} = \begin{bmatrix} Q - \Delta(\boldsymbol{\lambda}) & \Delta(\boldsymbol{\lambda}) & 0 & 0 & \cdots \\ \Delta(\boldsymbol{\mu}) & Q - \Delta(\boldsymbol{\lambda} + \boldsymbol{\mu}) & \Delta(\boldsymbol{\lambda}) & 0 & \cdots \\ 0 & \Delta(\boldsymbol{\mu}) & Q - \Delta(\boldsymbol{\lambda} + \boldsymbol{\mu}) & \Delta(\boldsymbol{\lambda}) & \cdots \\ 0 & 0 & \Delta(\boldsymbol{\mu}) & Q - \Delta(\boldsymbol{\lambda} + \boldsymbol{\mu}) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix},$$

where

$$\Delta(\boldsymbol{u}) = \Delta([u^{(1)}, u^{(2)}, \ldots, u^{(M)}]) = \begin{bmatrix} u^{(1)} & 0 & . & . & . & 0 \\ 0 & u^{(2)} & . & . & . & 0 \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ 0 & 0 & . & . & . & u^{(M)} \end{bmatrix}$$

The corresponding results regarding queue length probabilities are [NEUM81, p. 258]:

> Let $\boldsymbol{\pi}$ be the stationary probability vector of Q. Then the queue is *stable* if and only if
>
> $$\boldsymbol{\pi\lambda} < \boldsymbol{\pi\mu}.$$
>
> The matrix R is the minimal solution of the equation
>
> $$R^2\Delta(\boldsymbol{\mu}) + R[Q - \Delta(\boldsymbol{\lambda} + \boldsymbol{\mu})] + \Delta(\boldsymbol{\lambda}) = 0,$$
>
> and ... provided [the stability condition holds], we have
>
> $$R\boldsymbol{\mu} = \boldsymbol{\lambda}.$$

Also [NEUM81, p. 258]:

> The stationary probability vector $\boldsymbol{x} = [\boldsymbol{x}_0, \boldsymbol{x}_1, \ldots]$ of the stable queue is given by
>
> $$\boldsymbol{x}_k = \boldsymbol{\pi}(I - R)R^k, \textit{ for } k \geq 0.$$

The matrix R may be computed as follows [NEUM81, pp. 38, 258]:

1. $R(0) = 0$
2. $R(n+1) = \{[R(n)]^2\Delta(\boldsymbol{\mu}) + \Delta(\boldsymbol{\lambda})\}[\Delta(\boldsymbol{\lambda} + \boldsymbol{\mu}) - Q]^{-1}$
3. Repeat step 2 to determine R^* as the first $R(n+1)$ for which
   $$MAX_{i,j}\{R_{ij}(n+1) - R(n)\} < \epsilon$$
   for some small $\epsilon > 0$.
   This R^* is taken as the approximate value of R.

Waiting times in this queue are analyzed by considering a version of (N, E) in which no arrivals occur. The modified process begins with an arbitrary (random) number of customers and services them until the last customer departs, at which time the process is absorbed into one of the states of level 0. If the initial distribution of the number of customers present $\boldsymbol{y}(0)$ (as discussed previously) is chosen to be the stationary queue length distribution, $\boldsymbol{x}$, for (N, E) at arbitrary times, the time until absorption in the modified process is the virtual waiting time (the time that a virtual customer would spend waiting in the queue) for (N, E).

The generator of the absorbing process is

$$\tilde{Q}^0 = \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots \\ \Delta(\mu) & Q - \Delta(\mu) & 0 & 0 & \cdots \\ 0 & \Delta(\mu) & Q - \Delta(\mu) & 0 & \cdots \\ 0 & 0 & \Delta(\mu) & Q - \Delta(\mu) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix},$$

so that $A_2 = \Delta(\mu)$, $D = Q - \Delta(\mu)$, and $K = [Q - \Delta(\mu)]^{-1}\Delta(\mu)$. Thus, the vector of Laplace-Stieltjes transforms of the virtual waiting time distributions is

$$\boldsymbol{W}_V^*(s) = \boldsymbol{\pi}(I - R)\sum_{k=0}^{\infty} R^k[(sI + \Delta(\mu) - Q)^{-1}\Delta(\mu)]^k$$

and mean virtual waiting time is

$$\begin{aligned} \overline{W}_V &= -\boldsymbol{W}_V^{*\prime}(0)\mathbf{1} \\ &= \boldsymbol{\pi}(I - R)\sum_{k=1}^{\infty} R^k \sum_{\nu=0}^{k-1}[(Q - \Delta(\mu))^{-1}\Delta(\mu)]^{\nu}(Q - \Delta(\mu))^{-1}[(Q - \Delta(\mu))^{-1}\Delta(\mu)]^{k-\nu}\mathbf{1}. \end{aligned}$$

($\boldsymbol{W}_V^{*\prime}(s)$ denotes the vector of derivatives of $\boldsymbol{W}_V^*(s)$). Mean virtual time in the system (time in the queue plus service time) is thus

$$\overline{S}_V = \overline{W}_V + (\pi\mu)^{-1},$$

where $(\pi\mu)^{-1}$ is the mean *effective service time* [NEUM81, p. 272].

Other waiting times can be analyzed if the appropriate initial distributions for the $\tilde{Q}^0$ process are known. For example, if the vector $\boldsymbol{z} = [z_0, z_1, z_2, \ldots]$ of stationary queue length probabilities for the $\tilde{Q}$ process at arrival times is known, and $\boldsymbol{y}(0)$ is set equal to $\boldsymbol{z}$, then the time until absorption into a state of level 0 of $\tilde{Q}^0$ is the customer waiting time (not including service time) in $\tilde{Q}$.

3.2.2. Instantaneous Reconfigurations

Consider a single link j in a dynamic hierarchy. Let $E = \{E(t);\ t \geq 0\}$, a Markov process with finite state space $S_E = \{1, 2, , \ldots , M\}$ and irreducible generator

$$Q = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdot & \cdot & \cdot & \sigma_{1M} \\ \sigma_{21} & \sigma_{22} & \cdot & \cdot & \cdot & \sigma_{2M} \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ \sigma_{M1} & \sigma_{M1} & \cdot & \cdot & \cdot & \sigma_{MM} \end{bmatrix},$$

be the model of the external environment. Let $\pi = [\pi^{(1)}, \pi^{(2)}, \ldots , \pi^{(M)}]$. be the invariant probability vector of Q. (This process represents the environment of the entire network as well as that of each link. However, it is discussed here in terms of its effect on a single link).

Assume that on $\{E(t) = i\}$ the composite arrival process to link j is Poisson with rate $\lambda_j^{(i)}$, the length of a message entering service is reassigned according to an exponential distribution with mean μ^{-1}, the randomly chosen message lengths are independent, and the arrival and message length processes are independent. Let C_j be the transmission capacity of link j so that the service rate on link j every configuration is μC_j. Assume further that the time required to effect a reconfiguration is negligible. Then reconfigurations can be considered to occur instantaneously with environment state changes.

The following additional assumptions are included in this model:

- Waiting room (buffer space) available for messages queued at a link can be considered unlimited.
- Messages are transmitted, without interruption, on a first-come-first-served basis.
- Propagation times are negligible.

- Links are noiseless and error free.

This model is an RE-queue with arrival rate vector $\boldsymbol{\lambda}_j = [\lambda_j^{(1)}, \lambda_j^{(2)}, \ldots, \lambda_j^{(M)}]$ and service rate vector $\boldsymbol{\mu}_j = [\mu C_j, \mu C_j, \ldots, \mu C_j]$. The generator for the joint environment-queue length process is (N_j, E) is

$$\tilde{Q}_j = \begin{bmatrix} Q-\Delta(\boldsymbol{\lambda}_j) & \Delta(\boldsymbol{\lambda}_j) & 0 & 0 & \cdots \\ \mu C_j I & Q-(\Delta(\boldsymbol{\lambda}_j)+\mu C_j I) & \Delta(\boldsymbol{\lambda}_j) & 0 & \cdots \\ 0 & \mu C_j I & Q-(\Delta(\boldsymbol{\lambda}_j)+\mu C_j I) & \Delta(\boldsymbol{\lambda}_j) & \cdots \\ 0 & 0 & \mu C_j I & Q-(\Delta(\boldsymbol{\lambda}_j)+\mu C_j I) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}.$$

The queue is stable if and only if $\frac{1}{\mu C_j}\xi\boldsymbol{\lambda}_j < 1$.

Stationary queue length probabilities, mean queue length, and mean virtual waiting time for link j (in isolation) are computed by applying the previously discussed iterative method to determine R_j, the minimal solution to

$$R_j^2 \mu C_j I + R_j[Q-(\Delta(\boldsymbol{\lambda}_j)+\mu C_j I)] + \Delta(\boldsymbol{\lambda}_j) = 0,$$

and then computing the necessary matrix-geometric and related quantities.

3.2.2.1. Reconfiguration Periods

Consider now the following operating characteristic of the dynamic hierarchy: The arrival and message length processes are influenced by an external environment process as before. Additionally, preceding each reconfiguration, the network enters a reconfiguration state for a time interval known as a *reconfiguration period* [BHAU86] (See also [NAGS86]). Such a period allows the network nodes time to perform any processing necessary to enable operation in the new configuration. Let A_N be the set of potential apex nodes and A_L be the set of links connecting members of A_N. Then during a reconfiguration period, message transmission on links in A_L ceases. Messages arriving to a node in A_N (including external arrivals) intended for transmission over a link in A_L are queued at the link until the end of the reconfiguration period. Transmission on links not in A_L continues. Note that a link connecting a node in A_N and a node not in A_N is not in A_L. Thus, message transmission on such a link continues during a reconfiguration period.

To model this behavior, first the states $1_0, 2_0, \ldots, M_0$ are added to S_E to form the state space S_{E_1} of the environment process E_1. The ordering 1_0, 1, $2_0, \ldots, M_0$, M is imposed on the elements of S_{E_1}. When an environment transition from state $h \in \{1, 2,..., M\} = S_E$ occurs, instead of entering state $i \in S_E$, the environment enters state $i_0 \in \{1^0, 2^0, \ldots, M^0\} = S_E^0$. Simultaneously, the network enters configuration i_0, which is identical to configuration i, but in which operation follows the previously described reconfiguration procedure. The interval in which the network configuration is i_0 corresponds to a reconfiguration period. During this period, messages arrive from outside the network according to Poisson processes with rates identical to those induced by environment i.

Immediately following a period of residency in state i_0, the environment enters, with probability one, state i. The end of the reconfiguration period coincides with this transition. Normal network operation in configuration i proceeds. Arrivals from outside the network occur according to Poisson processes with rates induced by environment i.

For each link j not in A_L, regardless of whether the environment is in a *reconfiguration state* $i_0 \in S_E^0$ or a *normal state* $i \in S_E$, the length of a message entering service at a link is assumed to be reassigned according to an exponential distribution with mean μ^{-1}. The service rate at a link j with capacity C_j thus remains μC_j for every configuration. A similar assumption holds for links in A_L with the exception that the service rates on these links are zero when the environment is in a reconfiguration state.

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The independence assumptions of the previous link model are retained in this variant. That is, message lengths at link j are independent and given the state of the environment, the composite arrival process and the message length process are independent.

The remaining conventional assumptions (unlimited buffer space, FCFS service, negligible propagation times, and noiseless, error free links) are retained also. Further, it is assumed that a message being transmitted on a link in A_L when service is interrupted for a reconfiguration period is placed at the front of the queue and retransmitted in full when normal service resumes.

Let $E_1 = \{E_1(t);\ t \geq 0\}$, the environment process, be a Markov process with state space $S_{E_1} = \{1_0, 1, 2_0, \ldots, M_0, M\}$ and irreducible generator Q_1. Q_1 has the following nondiagonal elements:

$$
\begin{aligned}
&Q_1(h, i) = 0,\ h,\ i \in S_E,\\
&Q_1(h, i_0) = \sigma_{hi_0},\ h \in S_E,\ i_0 \in S_E^0,\\
&Q_1(h_0, i) = 0,\ h_0 \in S_E^0,\ i \in S_E,\ i \neq h,\\
&Q_1(h_0, h) = \sigma_{h_0h}.
\end{aligned}
$$

Thus, as previously described, from any normal state h, the environment may enter (in one step) any reconfiguration state and from any reconfiguration state h_0, it may enter only the associated normal state h.

Note that $Q_1(h, h_0)$ may be nonzero. Hence, state h may be followed in two steps by itself. The effect on the network of such a sequence of transitions is that it remains in configuration h and resumes normal operation after a reconfiguration period (Recall that configuration h_0 is, by definition, identical to configuration h).

The environment influences a link j as follows: On $\{E_1(t) = i\}$ for $i \in S_E$, the composite arrival process to link j is Poisson with rate $\lambda_j^{(i)}$ and the length of a message entering service is reassigned according to an exponential distribution with mean μ^{-1}. On $\{E_1(t) = i_0\}$ for $i_0 \in S_E^0$, the composite arrival process to link j is Poisson with rate $\lambda_j^{(i_0)}$. If link j is in A_L, no messages are transmitted (the service rate is zero). Otherwise, transmission proceeds as before at rate μC_j.

This model of a dynamic hierarchy link is an RE-queue with server interruptions. If $j \in A_L$, then (N_j, E_1) has arrival rate vector $\boldsymbol{\lambda}_j = [\lambda_j^{(1_0)}, \lambda_j^{(1)}, \lambda_j^{(2_0)}, \lambda_j^{(2)}, \ldots, \lambda_j^{(M_0)}, \lambda_j^{(M)}]$ and service rate vector $\boldsymbol{\mu}_j = [0, \mu C_j, 0, \mu C_j, \ldots, 0, \mu C_j]$. For j not in A_L, $\boldsymbol{\lambda}_j = [\lambda_j^{(1_0)}, \lambda_j^{(1)}, \lambda_j^{(2_0)}, \lambda_j^{(2)}, \ldots, \lambda_j^{(M_0)}, \lambda_j^{(M)}]$ and $\boldsymbol{\mu}_j = [\mu C_j, \mu C_j, \mu C_j, \mu C_j, \ldots, \mu C_j, \mu C_j]$. The $\lambda_j^{(i)}$ for $i \in S_E$ are the same as those of the previous model. However, the effect of service interruptions on links in A_L propagates throughout the network so that in general, $\lambda_j^{(i_0)} \neq \lambda_j^{(i)}$ for all i_0 irrespective of whether $j \in A_L$.

The generator for (N_j, E_1) is

$$\tilde{Q}_j = \begin{bmatrix} Q_1 - \Delta(\boldsymbol{\lambda}_j) & \Delta(\boldsymbol{\lambda}_j) & 0 & 0 & \cdots \\ \Delta(\boldsymbol{\mu}_j) & Q_1 - \Delta(\boldsymbol{\lambda}_j + \boldsymbol{\mu}_j) & \Delta(\boldsymbol{\lambda}_j) & 0 & \cdots \\ 0 & \Delta(\boldsymbol{\mu}_j) & Q_1 - \Delta(\boldsymbol{\lambda}_j + \boldsymbol{\mu}_j) & \Delta(\boldsymbol{\lambda}_j) & \cdots \\ 0 & 0 & \Delta(\boldsymbol{\mu}_j) & Q_1 - \Delta(\boldsymbol{\lambda}_j + \boldsymbol{\mu}_j) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}.$$

The stability condition for this queue is $\dfrac{\pi \boldsymbol{\lambda}_j}{\pi \boldsymbol{\mu}_j} < 1$, where

$\pi = [\pi^{(1_0)}, \pi^{(1)}, \pi^{(2_0)}, \ldots, \pi^{(M_0)}, \pi^{(M)}]$ is the invariant probability vector of Q_1.

As with the previous model, an appropriate matrix quadratic equation may be written and its minimal solution R_j used to compute stationary queue length probabilities, mean queue length, and mean virtual waiting time.

3.2.2.2. Other Models

In separate but related work, Bhat and Nance [BHAU86] propose a third model for links (and nodes) in the dynamic hierarchy. Each link is modeled as an independent M/M/1 queue and three periods of operation are considered: *normal operation, reconfiguration,* and *adjustment.* A reconfiguration period is a period during which the network configuration changes and transmission between potential apex nodes ceases. An adjustment period is the first busy period of a link (particularly a link between two potential apex nodes) following a reconfiguration period. Mean delay for a link is derived by combining the components due to each of these periods. Mean nodal delay is derived similarly.

A primary advantage of this approach is that it enables examination of of the busy period behavior of links (and nodes) following reconfigurations and the impact of this adjustment behavior on network performance. The models described here do not recognize adjustment periods explicitly. An adjustment period is included in the period of normal operation following a reconfiguration but is not analyzed separately. This approach, using RE-queue models, is better suited for examining steady-state network characteristics and the effects over time of a variable environment on the network.

4. Conclusions and Future Research

Of the two link models described here, the RE-queue with reconfiguration periods is the more realistic. It models the interaction between a variable environment (with a consequent variable set of demands) and a network link and recognizes the physical characteristic that a reconfiguration takes a nonzero amount of time.

The RE-queue with instantaneous reconfigurations is useful for modeling the basic dynamic nature of a link in a network with a dynamic topology. Further, its analysis

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should provide bounds on performance in the counterpart with nonzero length configuration periods. In particular, as the means of the reconfiguration period lengths approach zero, mean delay in the model with reconfiguration periods should approach mean delay in the models with instantaneous reconfigurations.

Work is currently in progress on an approximate network model of the dynamic hierarchy. A flow process approximation enables the network links to be analyzed separately through the use of the RE-queue models presented in this paper. The single-server RE-queue results are combined to produce approximate network performance measures. Simulation experiments will be conducted and their results used to assess the impact of the flow process approximation and the accuracy of the resultant performance measures.

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Dynamic hierarchical networks represent an architectural strategy for employing adaptive behavior in applications sensitive to highly variable external demands or uncertain internal conditions. The characteristics of such architectures are described, and the significance of adaptive capability is discussed. The necessity for assessing the tradeoffs between performance improvements (reduced end-to-end message transmission time, increased throughput) and the added costs (reconfiguration delays, redundant links, etc.) leads to the use of complex queueing models. The assumptions underlying the

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general model are stated, and a class of applicable models (queues in random environments or RE-queues) is introduced. Matrix-geometric methods are reviewed in terms of their suitability for addressing several variations of a subclass of RE-queue models. Matrix-geometric techniques are considered to offer the greatest promise for obtaining usable results for assessing the cost/benefit tradeoffs.